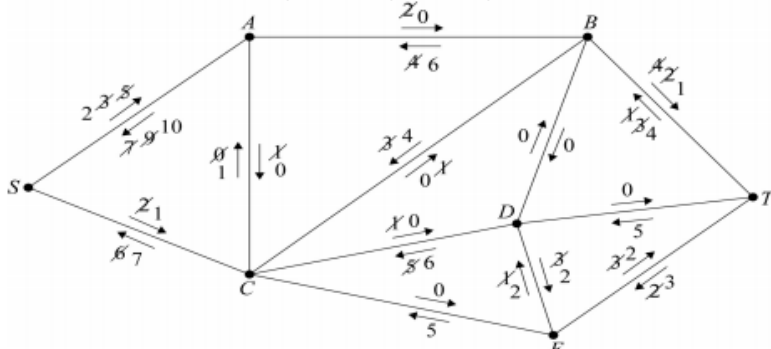
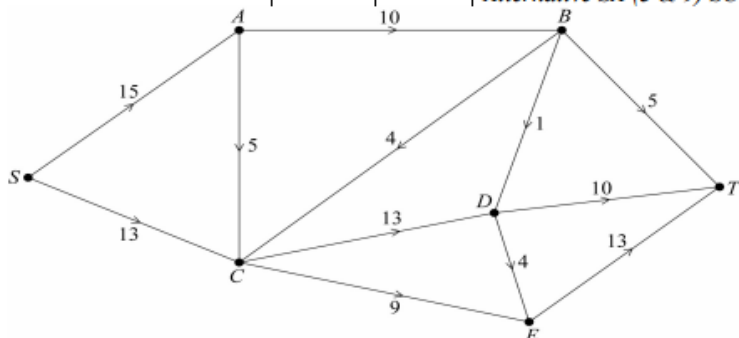
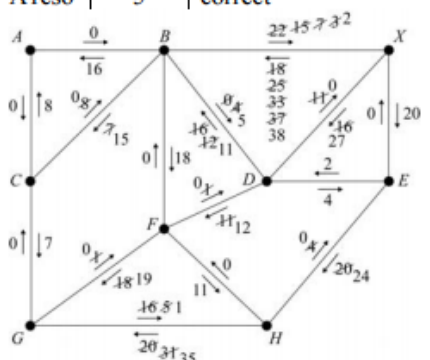
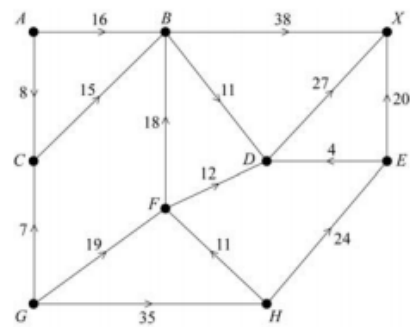


Q	Solution	Marks	Total	Comments								
6(a)	Value of cut = $10 + 10 + 15 - 4 - 1 = 30$	M1 A1	2	condone one slip if working shown								
(b)	$BT\ 2, DE\ 3, ET\ 12$	B1 B1	2	any 2 correct all correct								
(c)(i)	Initial flows forward and back or double Arc with arrows (at least 6 pairs correct)	M1 A1	2	Condone pairs of values, (coordinates) with single arrow all correct (condone pairs with single arrow provided key indicated)								
(ii)	<table border="1"> <thead> <tr> <th>Path</th> <th>Flow</th> </tr> </thead> <tbody> <tr> <td>SABT</td> <td>2</td> </tr> <tr> <td>SCDET</td> <td>1</td> </tr> <tr> <td>SACBT</td> <td>1</td> </tr> </tbody> </table> <p>(or SCBT instead of SACBT with flow 1)</p>	Path	Flow	SABT	2	SCDET	1	SACBT	1	M1 A1 A1		first correct path and flow another correct path and flow all correct (other possibilities also)
Path	Flow											
SABT	2											
SCDET	1											
SACBT	1											
(iii)	 <p>Must have forward and backward flows</p>	M1 A1	5	augmenting flows (6 pairs correct) correct Alternative SA (3 & 9) SC (0&8)								
(d)	 <p>May have SA(14), SC(14) and AC(4) using alternative Maximum flow values</p>	M1 A1 B1 B1	2	at least 8 correctly interpreted from their Figure 4 but $24 < \text{their maxflow} < 29$ <b>But</b> must have total flow of 28 in their network (condone one slip)								

Q	Solution	Marks	Total	Comments
6(a)	$SP \geq 12$ $SQ \geq 10$ $SR \geq 17$	B1	2	$S$ in correct place, (arrows) and capacities
	$YT \geq 18$ $ZT \geq 17$	B1		$T$ in correct place, (arrows) and capacities
(b)	$SPUYT$ 10	B1	2	
	$SRVWZT$ 8	B1		
(c)(i)	Initial flow forward and backward		2	withhold one B1 if paths to $S$ and $T$ not updated
	$PU$ 2 and 10 ; $UY$ 0 and 10 $RV$ 0 and 8 ; $VW$ 1 and 8 ; $WZ$ 2 and 8	B1 B1		
(ii)	Two correct routes and flows on Figure 6	M1	4	edges $UY$ , $UX$ , $WX$ and $WZ$ will be saturated $XY + XZ = 13$ in back flow
	Correct additional flows Max flow = 33	A1		
	Adjustment of at least 4 edges corresponding to flows (forward and backward)	M1		
	Correct final flows forward and backward (must score A1 for table)	A1 also		
(d)	Cut with value 33 is through $UY$ , $UX$ , $WX$ and $WZ$	B1	1	
	<b>Total</b>		<b>11</b>	
	<b>TOTAL</b>		<b>75</b>	

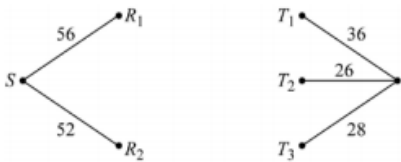
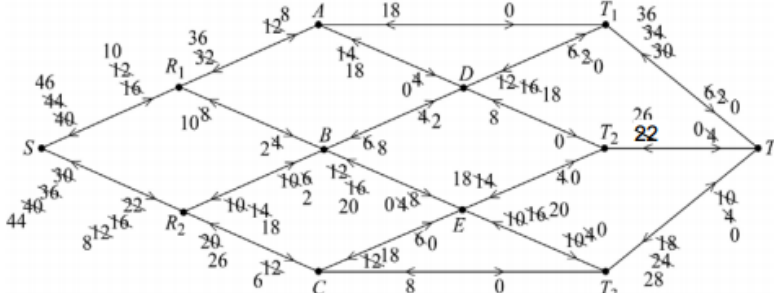
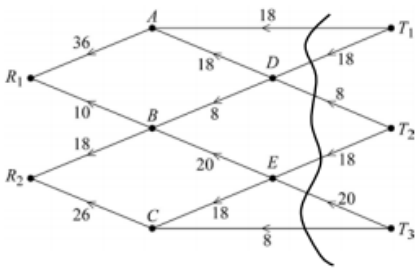
<p><b>5(a)</b></p>	<p>Cut value = <math>40 + 27 + 0 + 24</math> = 91</p>	<p>B1</p>	<p>1</p>																			
<p><b>(b)</b></p>	<p><i>ABDX</i> 16 <i>GFBX</i> 18 <i>GHEX</i> 20</p>	<p>B1 B1 B1</p>	<p>3</p>																			
<p><b>(c)(i)</b></p>	<p>One correct route with additional flow Another 2 routes and flows correct All routes correct with total flow = 85 Forward and backward flows on diagram (directions must be clear) Augmenting flows <i>Consider other possible correct flows</i> <i>Condone diagram as shown but really should have initial flows in DE, etc</i></p>	<p>M1 A1 A1 cso M1 A1 cso</p>	<p>5</p>	<p>any feasible route and flow total flow at least 80 at least 8 edges with pairs of values 'correct' correct</p>  <table border="1" data-bbox="1204 772 1396 1041"> <thead> <tr> <th>Route</th> <th>Flow</th> </tr> </thead> <tbody> <tr> <td><i>ABDX</i></td> <td>16</td> </tr> <tr> <td><i>GFBX</i></td> <td>18</td> </tr> <tr> <td><i>GHEX</i></td> <td>20</td> </tr> <tr> <td><i>GCBX</i></td> <td>7</td> </tr> <tr> <td><i>ACBX</i></td> <td>8</td> </tr> <tr> <td><i>GHFDX</i></td> <td>11</td> </tr> <tr> <td><i>GHEDBX</i></td> <td>4</td> </tr> <tr> <td><i>GFDBX</i></td> <td>1</td> </tr> </tbody> </table>	Route	Flow	<i>ABDX</i>	16	<i>GFBX</i>	18	<i>GHEX</i>	20	<i>GCBX</i>	7	<i>ACBX</i>	8	<i>GHFDX</i>	11	<i>GHEDBX</i>	4	<i>GFDBX</i>	1
Route	Flow																					
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<p><b>(ii)</b></p>	<p>Max flow = 85 Correct max flow <i>Consider other possible correct flows</i></p>	<p>B1 B1</p>	<p>2</p>																			
<p><b>(d)</b></p>	<p>Considering 'their' <math>AB+CB+FB - 45</math> = 4 fewer <math>\Rightarrow</math> max number = 81</p>	<p>M1 A1 cao</p>	<p>2</p>																			
<b>Total</b>			<b>13</b>																			

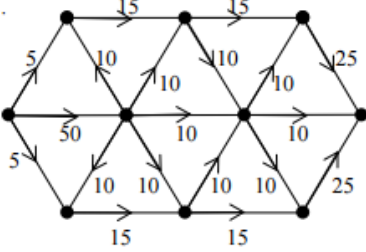
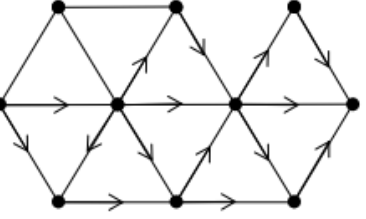
Q	Solution	Marks	Total	Comments																																																						
6(a)	$10 + 13 - 1 + 17$ $= 39$	M1 A1	2	3 values added and -1 (condone one slip)																																																						
(b)(i)	DE 12 FG 7	B1 B1	2	on Figure 2																																																						
(ii)	<table border="1"> <thead> <tr> <th>arc</th> <th>forward</th> <th>backward</th> </tr> </thead> <tbody> <tr><td>SA</td><td>3</td><td>1</td></tr> <tr><td>AB</td><td>1</td><td>1</td></tr> <tr><td>BT</td><td>0</td><td>1</td></tr> <tr><td>SC</td><td>0</td><td>2</td></tr> <tr><td>CA</td><td>0</td><td>1</td></tr> <tr><td>AD</td><td>0</td><td>1</td></tr> <tr><td>CD</td><td>1</td><td>1</td></tr> <tr><td>DE</td><td>1</td><td>2</td></tr> <tr><td>BE</td><td>1</td><td>3</td></tr> <tr><td>ET</td><td>2</td><td>3</td></tr> <tr><td>SF</td><td>1</td><td>1</td></tr> <tr><td>FC</td><td>1</td><td>2</td></tr> <tr><td>FD</td><td>1</td><td>0</td></tr> <tr><td>FG</td><td>0</td><td>1</td></tr> <tr><td>DG</td><td>2</td><td>1</td></tr> <tr><td>EG</td><td>1</td><td>1</td></tr> <tr><td>GT</td><td>2</td><td>3</td></tr> </tbody> </table>	arc	forward	backward	SA	3	1	AB	1	1	BT	0	1	SC	0	2	CA	0	1	AD	0	1	CD	1	1	DE	1	2	BE	1	3	ET	2	3	SF	1	1	FC	1	2	FD	1	0	FG	0	1	DG	2	1	EG	1	1	GT	2	3	M1		at least 6 pairs correct on Figure 3 (must have arrows)
arc	forward	backward																																																								
SA	3	1																																																								
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(iii)	<b>Table</b> <table border="1"> <thead> <tr> <th>Path</th> <th>Extra Flow</th> </tr> </thead> <tbody> <tr><td>SABET</td><td>1</td></tr> <tr><td>SFDGT</td><td>1</td></tr> <tr><td>SACDGT</td><td>1</td></tr> </tbody> </table>	Path	Extra Flow	SABET	1	SFDGT	1	SACDGT	1	M1 A1		1 correct path and extra flow all correct																																														
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SFDGT	1																																																									
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	<b>Network</b> 																																																									
(c)(i)	Max flow = 37	M1 A1 B1	4 1	1 path correctly augmented forward and backward <b>but</b> must have earned M1 in part (b)(ii) network correct																																																						
Q	Solution	Marks	Total	Comments																																																						
6(c) cont. (ii)	Max flow 	B2	2	correct flow of 37 condone 2 slips or omissions in flow of 37 <b>or</b> "correct" feasible flow of 36 for SC1																																																						
(d)	Cut through AB, AD, CD, FD and FG	B1	1	{ S, A, C, F } { B, D, E, G, T }																																																						
	<b>Total</b>		<b>14</b>																																																							
	<b>TOTAL</b>		<b>75</b>																																																							

Q	Solution	Marks	Total	Comments						
6(a)(i)	18 + (0+) 10 + 3 + 5 (= 36)	B1	1							
(ii)	30, 32, 36 (missing cut values)	B3	3	B1 each value correct						
(iii)	Max flow = 29 because value of minimum cut is 29	B1 E1	2	Award B0 E1 if their min <sup>m</sup> cut is < 29 and min <sup>m</sup> value explained as max flow						
(iv)		B1 cao	1	may have see alternative solution on next page						
(b)(i)		M1		potential flow (forward and back) 4 pairs 'correct' including SC and AT fit their (a)(iv) provided 0 < flow < 30						
		A1✓		all pairs correct (condone missing 0s) fit their (a)(iv) if correct flow < 29						
		m1		one correct flow in table						
		A1		table correct						
				<table border="1"> <tr> <td>SAT</td> <td>1</td> </tr> <tr> <td>SCT</td> <td>1</td> </tr> <tr> <td>SCBAT</td> <td>2</td> </tr> </table>	SAT	1	SCT	1	SCBAT	2
	SAT	1								
SCT	1									
SCBAT	2									
	m1		If (a)(iv) flow < 29 then may score A1 for correct table giving max flow of 33  (see also the alternative solution)							
			modifying flows (forward and back) 1 flow correct fit their initial flow							
		A1	6	modified flows all correct, including all 0s (may score A1 from a correct flow < 29 seen in (a)(iv) if final flow correct)						
(ii)	new max flow = 33	B1								
		M1		6 flows correctly interpreted from their labelling procedure provided M2 or M3 scored in (b)(i) (may have AB 2, AT 16, BT 9 – see over)						
		A1	3	flow correct SC B1 if flow of 33 shown correctly but not from correct labelling procedure						
<b>Total</b>			<b>16</b>							

Q	Solution	Marks	Total	Comments
(4)(a)(i)	Max Flow = 50 (Min cut = 50)	E1		Either statement
(ii)	$35 \leq \text{max flow} \leq 50$ (or min cut)	E1, E1		E1 for strict inequalities
(iii)	Error or contradiction	E1	4	oe
(b)	At F, $\left. \begin{array}{l} \text{flow in} \geq 8 \\ \text{flow out} \leq 7 \end{array} \right\}$	M1 A1	2	Stating F and one of the 'flows'
<b>Total</b>			<b>6</b>	
Q	Solution	Marks	Total	Comments
(8)(a)	ABEH 8 ACFH 5 ADGH 11	B1	1	
(b)(i)	ACEH 2 ACGH 4 Either ADFH 1 and ABFH 2  Or ADFH 3	M1 A1 A1		One correct route and flow At least one other correct All correct
		M1  A1	5	Forward and back flows on diagram  All correct
(ii)	Max flow 33	B1		
		B1	2	OE
(c)	Cut through BE, CE, FH, CG, DG	B1	1	
<b>Total</b>			<b>9</b>	

<b>2(a)</b>	19	B1	1	
<b>(b)</b>	E	B1	1	
<b>(c)</b>	C	B1	1	
<b>(d)</b>	$x = 8$ $y = 13$ $z = 39$	$B1 \times 3$	3	
<b>(e)</b>	76	B1	1	
<b>(f)</b>	83	B1	1	
<b>Total</b>			<b>8</b>	

<b>7(a)</b>		B1 B1	2	Edges with values $\geq 56, 52$ Edges with values $\geq 36, 26, 28$
<b>b(i)</b>		M1 A1 M1 A1 A1	5	initial diagram with forward/back flows Fully correct diagram One correct path and flow At least one other correct path and flow all correct (ignore connections to S and T)
<b>(ii)</b>	<p>Max flow 90</p> 	B1	2	
<b>(c)</b>	<p>Cut through (shown)</p> <p><math>AT_1, DT_1, DT_2, ET_2, ET_3, CT_3</math></p>	B1 B1	2	PI by correct list OE
<b>Total</b>			<b>11</b>	

Q	Solution	Mark	Total	Comment
<b>3(a)</b>	$C_1 = 60$ $C_2 = 80$	<b>B1</b> <b>B1</b>	<b>2</b>	
<b>(b)</b>	e.g. 	<b>M1</b> <b>A1</b>	<b>2</b>	Correct at <i>D</i>
<b>(c)(i)</b>		<b>M1</b> <b>A1</b>		Correct to <i>D, E, F</i> either by inspection or flow augmentation All correct
	oe		<b>3</b>	
	MAX = 45	<b>B1</b>	<b>3</b>	
<b>(ii)</b>	CUT THRU' <i>EG, DG, DF, DC, AC</i> Max flow = Min cut	<b>B1</b> <b>E1</b>	<b>2</b>	Or <i>{A,B,D,E} {C,FG,,H,IJ}</i> Must have scored B1,B1 in point (C)
	<b>Total</b>		<b>9</b>	